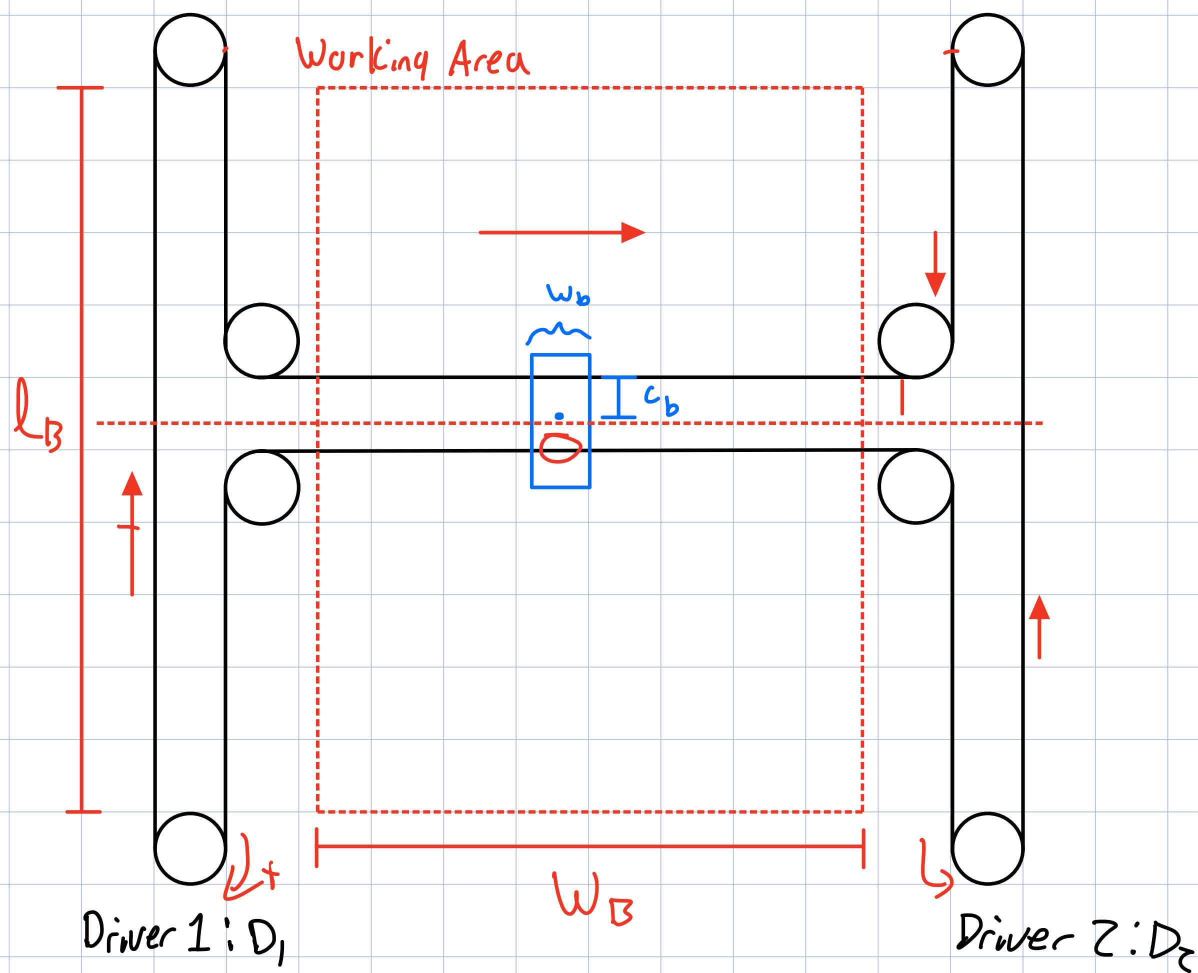
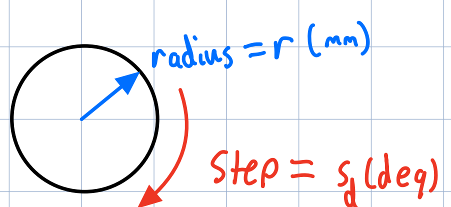


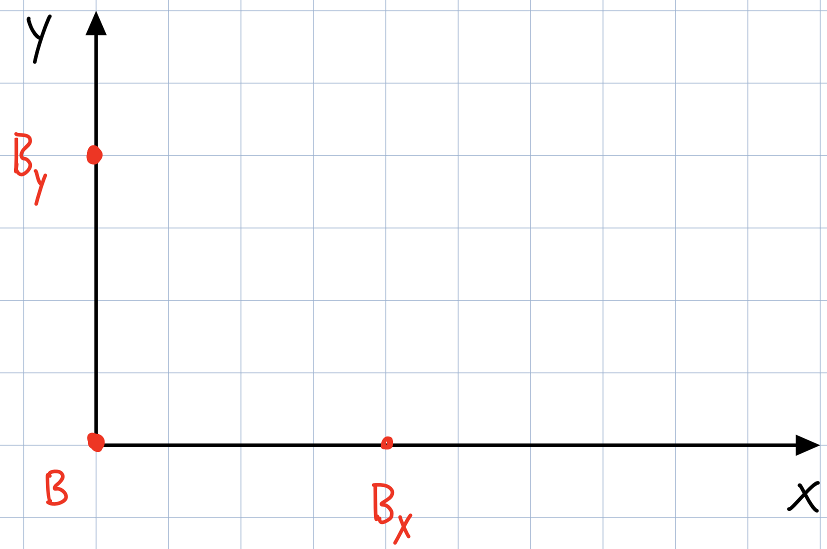
Kinematics

Driver Pulley



$$S_L = \text{Length per step} = \frac{s_d}{360\left(\frac{deg}{rad}\right)} \cdot 2\pi r$$

2 Scenarios - Lateral movement, Diagonal movement



$B \rightarrow B_x$: Horizontal movement

$$\theta_{D_1} = \theta_{D_2}$$
$$\dot{\theta}_{D_1} = \dot{\theta}_{D_2}$$

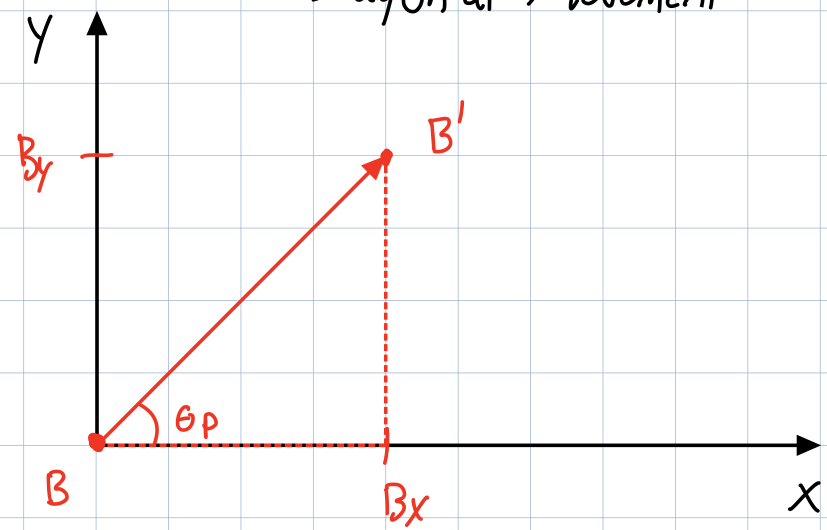
$$X = \#\text{ of steps} = \frac{B_x - B}{S_L}$$

$B \rightarrow B_y$: vertical movement

$$\theta_{D_1} = -\theta_{D_2}$$
$$\dot{\theta}_{D_1} = -\dot{\theta}_{D_2}$$

$$Y = \#\text{ of steps} = \frac{B_y - B}{S_L}$$

Diagonal Movement



i) If $B_x > 0$ $B_y > 0$, $\dot{\theta}_{D_1} < 0$ $\dot{\theta}_{D_2} > 0$

$|\dot{\theta}_{D_1}|$ $|\dot{\theta}_{D_2}|$

If $B_x > 0$ $B_y < 0$, $\dot{\theta}_{D_1} > 0$ $\dot{\theta}_{D_2} < 0$

$|\dot{\theta}_{D_1}| > |\dot{\theta}_{D_2}|$

If $B_x < 0$ $B_y > 0$, $\dot{\theta}_{D_1} < 0$ $\dot{\theta}_{D_2} > 0$

$|\dot{\theta}_{D_1}| > |\dot{\theta}_{D_2}|$

If $B_x < 0$ $B_y < 0$, $\dot{\theta}_{D_1} < 0$ $\dot{\theta}_{D_2} > 0$

$$|\dot{\theta}_{D_1}| < |\dot{\theta}_{D_2}|$$

Code outline

B_{x_1} = x coordinate - initial position
B_{y_1} = y coordinate - initial position

B_{x_2} = Designated x-coordinate
B_{y_2} = Designated y-coordinate

$D_x = B_{x_2} - B_{x_1}$
$D_y = B_{y_2} - B_{y_1}$

Lateral Movement

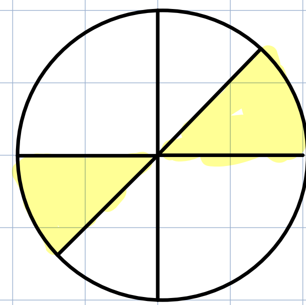
If $D_x = 0$ & $D_y > 0$
$\dot{\theta}_1 < 0$ $\dot{\theta}_2 > 0$

If $D_x > 0$ & $D_y = 0$
$\dot{\theta}_1 > 0$ & $\dot{\theta}_2 > 0$

If $D_x = 0$ & $D_y < 0$
$\dot{\theta}_1 > 0$ $\dot{\theta}_2 < 0$

If $D_x < 0$ & $D_y = 0$
$\dot{\theta}_1 < 0$ & $\dot{\theta}_2 < 0$

Diagonal Movement



Kinematics when $|\dot{\theta}_1| = max$

Each movement w/ fixed $|\dot{\theta}_1|$

$\dot{\theta}_1 < 0 \quad \dot{\theta}_2 < 0$

$\dot{\theta}_1 = constant \qquad \dot{\theta}_1 > 0 \quad \dot{\theta}_2 > 0 \quad |\dot{\theta}_1| > |\dot{\theta}_2|$

$$B_x = -\dot{\theta}_2 \cdot S_L \cdot t - \frac{1}{2}(\dot{\theta}_1 - \dot{\theta}_2) \cdot S_L \cdot t$$

$$B_x = -\frac{1}{2} S_L t (\dot{\theta}_2 + \dot{\theta}_1)$$

$$\dot{\theta}_2 = -\frac{2B_x}{S_L t} - \dot{\theta}_1$$

$$B_y = -\frac{1}{2}(\dot{\theta}_1 - \dot{\theta}_2) S_L \cdot t$$

$$t = -\frac{2B_y}{(\dot{\theta}_1 - \dot{\theta}_2) S_L}$$

$$\dot{\theta}_2 = \frac{2B_x (\dot{\theta}_1 - \dot{\theta}_2)}{2B_y} - \dot{\theta}_1$$

$$\dot{\theta}_2 = \dot{\theta}_1\left(\frac{B_x}{B_y} - 1\right) - \frac{B_x}{B_y}\dot{\theta}_2$$

$$\left(1+\frac{B_x}{B_y}\right)\dot{\theta}_2 = \dot{\theta}_1\left(\frac{B_x}{B_y} - 1\right)$$

$$\boxed{\dot{\theta}_2 = \dot{\theta}_1 \frac{\left(\frac{B_x}{B_y} - 1\right)}{\left(1 + B_x/B_y\right)}}$$ Quad 2/8

Steps

$$\int_0^t \dot{\theta}_2\,dt = \int_0^t \dot{\theta}_1 \frac{\left(\frac{B_x}{B_y} - 1\right)}{\left(1 + B_x/B_y\right)}\,dt$$

$$\theta_2 = \theta_1 \frac{\left(\frac{B_x}{B_y} - 1\right)}{\left(1 + B_x/B_y\right)}$$

$$B_x = -\tfrac{1}{2} S_L(\theta_2 + \theta_1)$$

$$\frac{-2B_x}{S_L} - \theta_2 = \theta_1$$

$$\theta_2 = \left(-\frac{2B_x}{S_L} - \theta_2\right)\left(\frac{\frac{B_x}{B_y} - 1}{1 + B_x/B_y}\right)$$

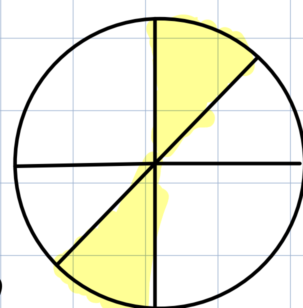
$$\theta_2 = \frac{-2B_x}{S_L} \cdot \left(\frac{\frac{B_x}{B_y} - 1}{1 + B_x/B_y}\right) - \theta_2\left(\frac{\frac{B_x}{B_y} - 1}{1 + B_x/B_y}\right)$$

$$\theta_2\left(1+\left(\frac{B_x/B_y-1}{1+B_x/B_y}\right)\right)=\frac{-2B_x}{S_L}\cdot\left(\frac{\frac{B_x}{B_y}-1}{1+B_x/B_y}\right)$$

$$\boxed{\theta_2=\frac{\frac{-2B_x}{S_L}\left(\frac{B_x/B_y-1}{1+B_x/B_y}\right)}{\left(1+\frac{(B_x/B_y-1)}{1+B_x/B_y}\right)}}$$

$$\boxed{\theta_1=\frac{\frac{-2B_x}{S_L}}{\left(1+\frac{(B_x/B_y-1)}{1+B_x/B_y}\right)}}$$

Kinematics when $|\dot{\theta}_1| = max$



Each movement w/ fixed $|\dot{\theta}_1|$

$\dot{\theta}_1 < 0 \quad \dot{\theta}_2 > 0$

$\dot{\theta}_1 = \text{constant} \qquad \dot{\theta}_1 > 0 \quad \dot{\theta}_2 < 0 \quad |\dot{\theta}| > |\dot{\theta}_2|$

$$B_y = \dot{\theta}_2 \cdot S_L \cdot t - \tfrac{1}{2}(\dot{\theta}_1 + \dot{\theta}_2)\cdot S_L \cdot t$$

$$B_y = \tfrac{1}{2} S_L t(\dot{\theta}_2 - \dot{\theta}_1)$$

$$\dot{\theta}_2 = \frac{2B_y}{S_L t} + \dot{\theta}_1$$

$$B_x = -\frac{1}{2}(\dot{\theta}_1 - \dot{\theta}_2) S_L \cdot t$$

$$t = -\frac{2B_x}{(\dot{\theta}_1 - \dot{\theta}_2) S_L}$$

$$\dot{\theta}_2 = -\frac{2B_y(\dot{\theta}_1 - \dot{\theta}_2)}{2B_x} + \dot{\theta}_1$$

$$\dot{\theta}_2 = \dot{\theta}_1\left(1 - \frac{B_y}{B_x}\right) - \frac{B_y}{B_x}\dot{\theta}_2$$

$$\left(1 + \frac{B_y}{B_x}\right)\dot{\theta}_2 = \dot{\theta}_1\left(1 - B_y/B_x\right)$$

$$\boxed{\dot{\theta}_2 = \dot{\theta}_1 \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)}}$$

From 90° or 270°

$$\int_0^t \dot{\theta}_2\,dt = \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)} \int_0^t \dot{\theta}_1\,dt$$

$$\theta_2 = \theta_1 \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)}$$

$$B_y = \tfrac{1}{2} S_L (\theta_2 - \theta_1)$$

$$\theta_1 = \left(\frac{-2B_y}{S_L} + \theta_2 \right)$$

$$\theta_2 = \left(\frac{-2B_y}{S_L} + \theta_2 \right) \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)}$$

$$\theta_2 = \frac{-2B_y}{S_L} \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)} + \theta_2 \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)}$$

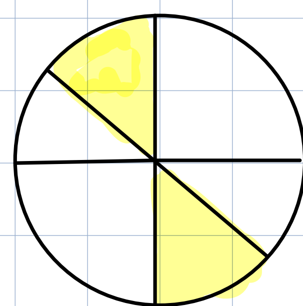
$$\theta_2 \left(1 - \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)} \right) = \frac{-2B_y}{S_L} \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)}$$

$$\boxed{\theta_2 = \frac{\dfrac{-2B_y}{S_L} \dfrac{(1 - B_y/B_x)}{(1 + B_y/B_x)}}{\left(1 - \dfrac{(1 - B_y/B_x)}{(1 + B_y/B_x)} \right)}}$$

$$\boxed{\theta_1 = \frac{\frac{-2B_y}{S_L}}{\left(1 - \frac{(1 - B_y/B_x)}{(1 + B_y/B_x)}\right)}}$$

Kinematics when $|\dot{\theta}_2| = max$

Each movement w/ fixed $|\dot{\theta}_2|$



$\dot{\theta}_2 = constant$

$\dot{\theta}_2 > 0$ $\dot{\theta}_1 < 0$ $\dot{\theta}_2 < 0$ $\dot{\theta}_1 > 0$

$|\dot{\theta}_2| > |\dot{\theta}_1|$

$$B_y = -\dot{\theta}_1 \cdot S_L \cdot t + \frac{1}{2}(\dot{\theta}_2 + \dot{\theta}_1) \cdot S_L \cdot t$$

$$B_y = \frac{1}{2} S_L t (\dot{\theta}_2 - \dot{\theta}_1)$$

$$\dot{\theta}_1 = \dot{\theta}_2 - \frac{2B_y}{S_L t}$$

$$B_x = -\frac{1}{2}(\dot{\theta}_1 + \dot{\theta}_2) S_L \cdot t$$

$$t = -\frac{2B_x}{(\dot{\theta}_1+\dot{\theta}_2)S_L}$$

$$\dot{\theta}_1 = \dot{\theta}_2 + \frac{B_y}{B_x}(\dot{\theta}_1+\dot{\theta}_2)$$

$$\dot{\theta}_1 = \dot{\theta}_1\frac{B_y}{B_x} + \dot{\theta}_2(1+B_y/B_x)$$

$$\dot{\theta}_1(1-B_y/B_x) = \dot{\theta}_2(1+B_y/B_x)$$

$$\boxed{\dot{\theta}_1 = \dot{\theta}_2\frac{(1+B_y/B_x)}{(1-B_y/B_x)}}$$

$$\int_0^t \dot{\theta}_1 = \int_0^t \dot{\theta}_2\frac{(1+B_y/B_x)}{(1-B_y/B_x)}$$

$$\theta_1 = \theta_2\frac{(1+B_y/B_x)}{(1-B_y/B_x)}$$

$$B_x = -\tfrac{1}{2}(\theta_1+\theta_2)S_L$$

$$\theta_2 = \frac{-2B_x}{S_L} - \theta_1$$

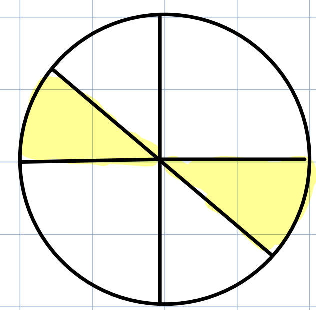
$$\Theta_1 = \left(-\frac{2B_x}{S_L} - \Theta_1\right)\frac{(1 + B_y/B_x)}{(1 - B_y/B_x)}$$

$$\Theta_1\left(1 + \frac{(1 + B_y/B_x)}{(1 - B_y/B_x)}\right) = -\frac{2B_x}{S_L}\frac{(1 + B_y/B_x)}{(1 - B_y/B_x)}$$

$$\boxed{\Theta_1 = \frac{-\frac{2B_x}{S_L}\frac{(1 + B_y/B_x)}{(1 - B_y/B_x)}}{\left(1 + \frac{(1 + B_y/B_x)}{(1 - B_y/B_x)}\right)}}$$

$$\boxed{\Theta_2 = \frac{-\frac{2B_x}{S_L}}{\left(1 + \frac{(1 + B_y/B_x)}{(1 - B_y/B_x)}\right)}}$$

Kinematics when $|\dot{\theta}_2| = max$



Each movement w/ fixed $|\dot{\theta}_2|$

$\dot{\theta}_1 > 0 \quad \dot{\theta}_2 > 0$

$\dot{\theta}_2 = constant \qquad \dot{\theta}_1 < 0 \quad \dot{\theta}_2 < 0 \quad |\dot{\theta}_1| < |\dot{\theta}_2|$

$$B_x = -\dot{\theta}_1 \cdot S_L \cdot t - \frac{1}{2}(\dot{\theta}_2 - \dot{\theta}_1) \cdot S_L \cdot t$$

$$B_x = -\frac{1}{2} S_L t (\dot{\theta}_2 + \dot{\theta}_1)$$

$$\dot{\theta}_1 = -\frac{2B_x}{S_L t} - \dot{\theta}_2$$

$$B_y = \frac{1}{2}(\dot{\theta}_2 - \dot{\theta}_1) S_L \cdot t$$

$$t = \frac{2B_y}{(\dot{\theta}_2 - \dot{\theta}_1) S_L}$$

$$\dot{\theta}_1 = -\frac{2B_x (\dot{\theta}_2 - \dot{\theta}_1)}{2B_y} - \dot{\theta}_2$$

$$\dot{\theta}_1 = -\dot{\theta}_2\left(\frac{B_x}{B_y} + 1\right) + \frac{B_x}{B_y}\dot{\theta}_1$$

$$\left(1 - \frac{B_x}{B_y}\right)\dot{\theta}_1 = -\dot{\theta}_2\left(\frac{B_x}{B_y} + 1\right)$$

$$\boxed{\dot{\theta}_1 = -\dot{\theta}_2 \frac{\left(\frac{B_x}{B_y} + 1\right)}{\left(1 - B_x/B_y\right)}}$$

$$\int_0^t \dot{\theta}_1 \, dt = \int_0^t -\dot{\theta}_2 \frac{\left(B_x/B_y + 1\right)}{\left(1 - B_x/B_y\right)}$$

$$\theta_1 = -\theta_2 \frac{\left(B_x/B_y + 1\right)}{\left(1 - B_x/B_y\right)}$$

$$B_y = 1/2\left(\theta_2 - \theta_1\right) S_L$$

$$\theta_2 = \frac{2B_y}{S_L} + \theta_1$$

$$\theta_1 = -\frac{2B_y}{S_L} \frac{\left(B_x/B_y + 1\right)}{\left(1 - B_x/B_y\right)} + \theta_1 \frac{\left(B_x/B_y + 1\right)}{\left(1 - B_x/B_y\right)}$$

$$\theta_1 \left(1 - \frac{\left(\beta x/\beta y + 1 \right)}{\left(1 - \beta x/\beta y \right)} \right) = \frac{-2\beta y}{SL} \frac{\left(\beta x/\beta y + 1 \right)}{\left(1 - \beta x/\beta y \right)}$$

$$\theta_1 = \boxed{\frac{\frac{-2\beta y}{SL} \frac{\left(\beta x/\beta y + 1 \right)}{\left(1 - \beta x/\beta y \right)}}{\left(1 - \frac{\left(\beta x/\beta y + 1 \right)}{\left(1 - \beta x/\beta y \right)} \right)}}$$

$$\boxed{\theta_2 = \frac{\frac{2\beta y}{SL}}{\left(1 - \frac{\left(\beta x/\beta y + 1 \right)}{\left(1 - \beta x/\beta y \right)} \right)}}$$