California Polytechnic State University - San Luis Obispo, CA 93407
Dept. of Mechanical Engineering

ME 518-03

Machinery Vibration and Rotor Dynamics

Winter 2023

Term Project

Student
Dylan Ruiz

Professor
Xi Wu

San Luis Obispo
03/23/2023

1. Abstract

The objective of this term project was to create and analyze a differential planetary gear transmission system. The gears' involute profile was calculated through MATLAB scripts for both internal and external configurations. Using SolidWorks and the calculated gear involute profiles, the gears were modeled and assembled. The gear models were saved and imported into ADAMS View to analyze the gear transmission system. Three cases were analyzed with the gear transmission system, a fixed ring gear, a fixed sun gear and no restraints to any of the gears. Theoretical calculations were applied with the use of MATLAB to ensure the accuracy of the ADAMS simulation results, specifically for the carrier speed.

2. Introduction

A planetary gearset, also known as an epicyclic gear train, is a gear system consisting of one or more "planet gears" that revolve around a central gear known as the "sun gear". The planet gears are mounted using a carrier which rotates relative to the sun gear. This gearing system also incorporates a "ring gear" which meshes with the planet gears. Figure 1 below shows an example of a planetary gearset that incorporates 4 planet gears(blue) mounted to a carrier(green) that revolves around the sun gear(yellow) and meshes with the ring gear (red).

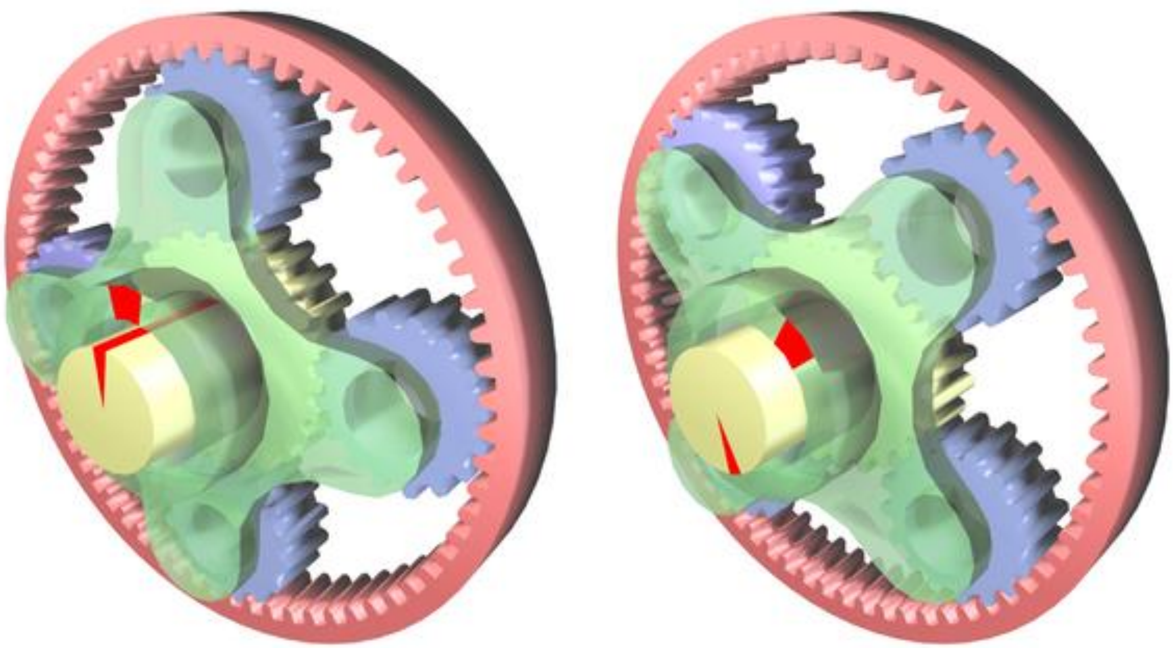


Figure 1: Planetary Gear Example

The differential planetary gear transmission system modeled for this term project utilizes a three planet gearset along with a gear pair "Z1" and "Z2" used to rotate the ring gear when it is not being fixed. Figure 2 shows the SOLIDWORKS model of this transmission system.

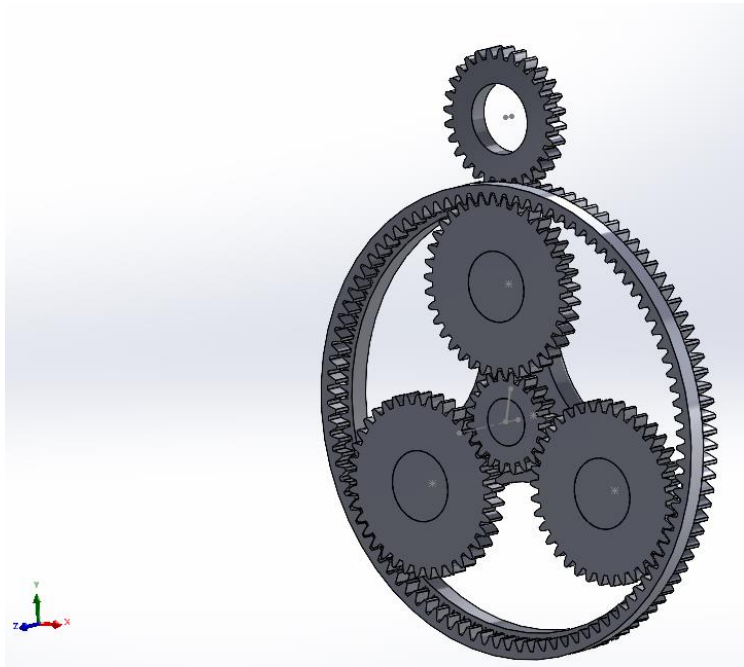


Figure 2: Differential Planetary Gear Transmission System CAD Model

3. SolidWorks Modeling

3.1 Generation of teeth profile

To begin modeling the gears the involute profile of the gear teeth is needed for each gear. The gear teeth are modeled based on the involute profile which maintains a constant pressure angle of 20°. The profile was calculated using a MATLAB script created by Dr. Wu and modified by Andrew Sommer and Nicholas D. Luzuriaga. This MATLAB script saves a text file that specifies the coordinates of the involute profile as well as gives the radii of the base, addendum, dedendum, and pitch circles. Figure 3 shows an example of the involute profile the MATLAB script outputs for the sun gear.

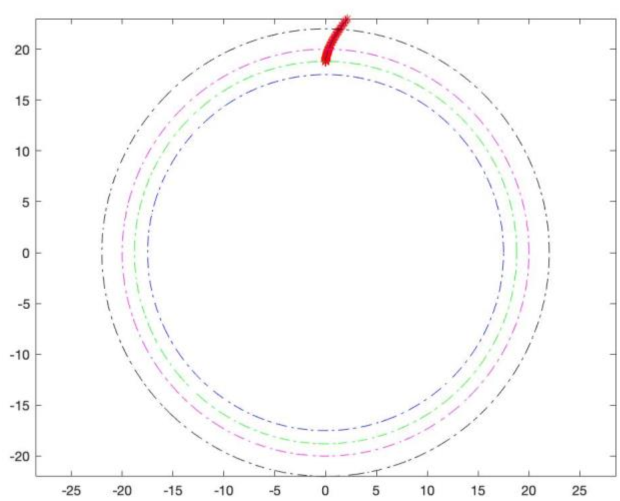


Figure 3: Involute Profile of the Sun Gear

After importing the text file into SolidWorks, it is required to shift the profile by the specified number of degrees. To ensure reasonable backlash, the involute profile was rotated to 98% of the specified amount.

3.2 SolidWorks Modeling

For each gear, the involute profile was imported into SolidWorks and shifted by the 98%. The profile was connected to the dedendum and cut at the addendum to create the general shape of one tooth. The tooth was then extruded along with the rest of the gear and patterned to create identical, equally space teeth along the gear. All gears in this project are specified to be 8mm thick, except for the Z2 which was specified at 9mm. This gear is thicker so the Z1 gear doesn't rub against the ring gear when meshing with the Z2 gear. Fillets were added to ensure better meshing between the gears. The carrier was modeled based on the size of the gears and included mounting points for each of the planet gears and the sun gear. After everything was assembled through SolidWorks assembly, the gears were mated using gear mates and analyzed to make sure meshing interference wasn't present. After tuning the gear teeth to ensure meshing occurs, the original suggestion of the thickness of the gear teeth at the base circle differed than the final result. *Table 1* shows the difference in these thicknesses.

Table 1: Base Circle Tooth Width

Gear	Tuned Base Circle Tooth Width (mm)	MATLAB Base Circle Tooth Width (mm)
Planet	3.786	3.988
Z1	3.659	3.736
Z2	5.700	5.697
Ring	4.933	5.585
Sun	3.300	3.512

After finalizing the SolidWorks models, the models were saved as parasolids file type and imported into Adams.

4. Adams Modeling

Before importing the SolidWorks models into ADAMS, the coordinates of each of the models in the SolidWorks assembly were noted and used to create markers in ADAMS. This way, once the gears are imported into ADAMS, they can easily be moved and positioned to the correct coordinates to ensure perfect meshing with each other. An isometric view of the ADAMS model can be viewed in *Figure 4.*

4.1 Model Setup: Joints

In order to ensure the gears, move correctly in relation to one another, the joints applied to each of the gears must be specified. Revolute joints for each of the gears must be applied to the right reference. Since the Z2 gear, Ring gear, and carrier rotate about the same axis, their revolute joint must be rotated about the same point, the geometric center of the carrier. The Z1 gear must only rotate about its own geometric center as it's rotating about a fixed mount. The planet gears must rotate in relation to the carrier's mounting points, so the revolute joint is referenced to the carrier. Finally, since the Z2 gear and the ring gear rotate at the same speed, a fixed joint must be applied between the two gears to ensure no slipping.

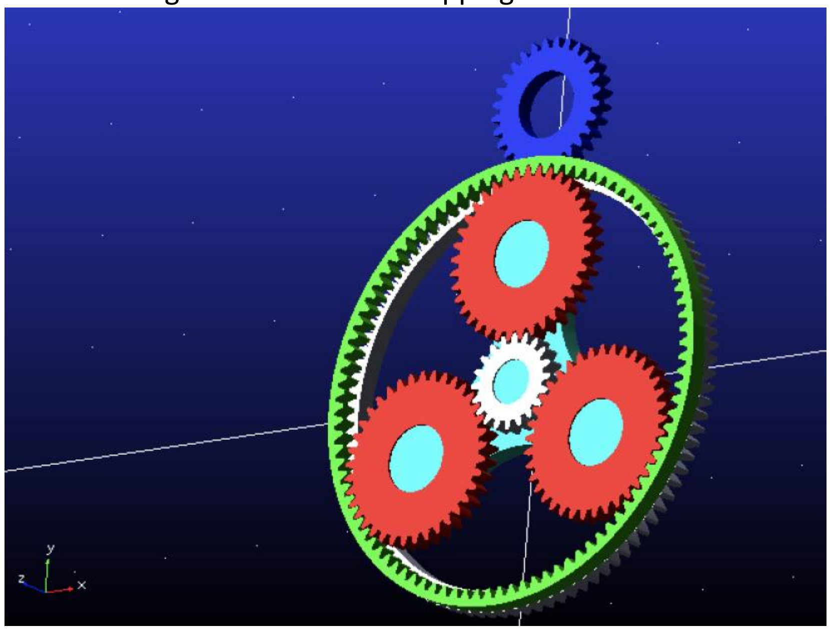


Figure 4: Isometric View of Adams Planetary Gear Model

4.2 Model Setup: Contact Forces

The interaction between the gears was simulated through the use of contact forces within the ADAMS simulation. Contact forces were applied between each of the gear pairs. Each of the contact forces used the same parameters shown in *Figure 5*.

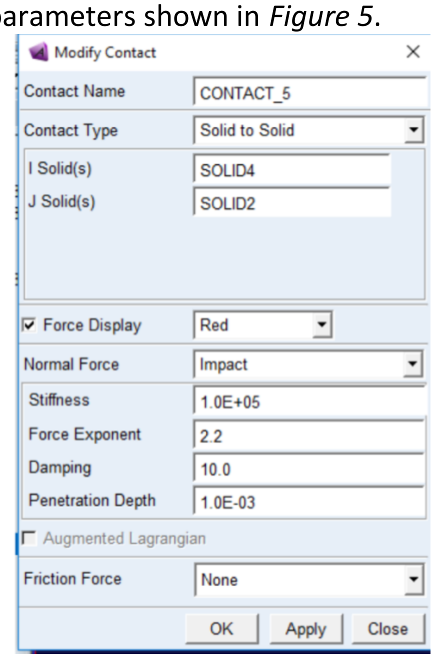


Figure 5: Contact Force Parameters

Choosing the impact Normal Force proved to be less erroneous when simulating the system. The restitution model simulates the meshing of the gears better but would never finish simulating due to some error that occurs during the simulation. This error was most likely due to the meshing of the gears. The Impact model proved to simulate fine without much fine-tuning to the parameters, however a very small time step during simulation was required for a successful run.

4.3 Model Setup: Motions

There was one motion applied to the system for all three of the cases. For the fixed sun case, the motion of 750 rpm was applied to the Z1 gear, which moves the ring and the planet gears around the sun. For the fixed ring case, the motion of 750 rpm was applied to the sun gear. This case analyzed the motion of the planet gears and carrier with respect to a fixed ring gear. Finally, the last case, with no fixed gears, was applied to the Z1 gear. This is the differential gear transmission case. To create a motion of 750 rpm, ADAMS required an input of 4500d * t, or 4500 degrees per second for the angular velocity. Since these speeds don't ramp up and happen immediately, the beginning of the simulation provides erroneous data due to the sudden ramp in angular velocity.

5. Simulation Results

The carrier speed was analyzed for each of the three cases. These speeds were compared to the theoretical speeds calculated using MATLAB.

5.1 Case 1: Fixed Sun Gear

The first case analyzed was the fixed sun gear case. The motion was applied to the Z1 gear and the carrier's angular velocity was analyzed with respect to time. *Figure 6* shows the plot of the first case.

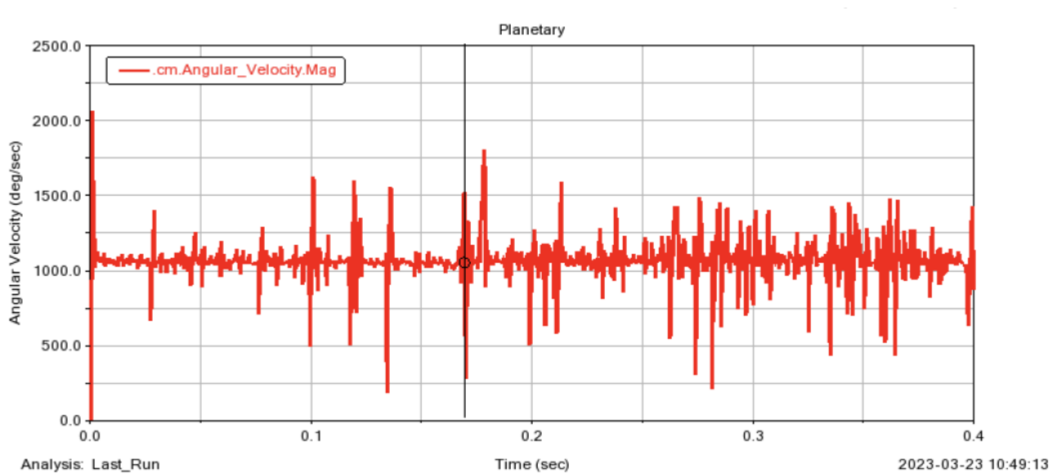


Figure 6: Case 1 Carrier Angular Velocity

As shown above, the angular velocity of the carrier was highly variable, oscillating around 1100 deg/s. The oscillations shown in the plot are most likely due to poor meshing of the gears. Since the ADAMS simulation constantly failed, reducing the width of each of the gear teeth was required to ensure a better clearance when meshing. Due to this reduction in size, the meshing of the gears reduced in quality and created a variable speed of the carrier. In the beginning of the plot, a big spike is shown due to the sudden ramp in angular velocity of the Z1 gear. The theoretical MATLAB calculation for the carrier speed for this case is 1060.2 deg/s. The MATLAB calculations is shown in **Appendix A**. This value lies within the region of oscillation for the carrier speed, so the simulation is validated.

5.3 Case 2: Fixed Ring Gear

The second case analyzed was the fixed ring gear case. The motion was applied to the sun gear and the carrier's angular velocity was analyzed with respect to time. *Figure 7* shows the plot of the second case.

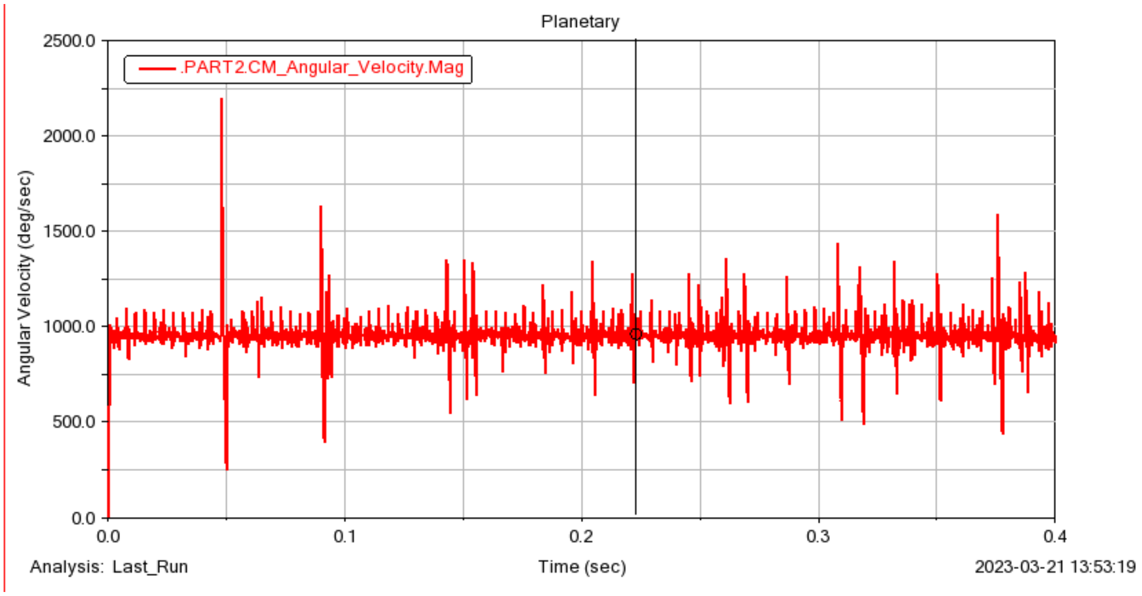


Figure 7: Case 2 Carrier Angular Velocity

As shown above, the angular velocity of the carrier was highly variable, oscillating just below 1000deg/s. Similar to the first case, the oscillations shown in the plot are most likely due to poor meshing of the gears. There are multiple spikes within the plot which shows the simulation encountered some sort of poor meshing between gears at that point in time. The theoretical MATLAB calculation for the carrier speed for this case is 957.5 deg/s. The MATLAB calculations is shown in **Appendix A**. This value lies within the region of oscillation for the carrier speed, so the simulation is validated.

5.3 Case 3: Gear Transmission System

The third case analyzed was the gear transmission system. The motion was applied to the Z1 gear and the carrier's angular velocity was analyzed with respect to time. *Figure 8* shows the plot of the third case.

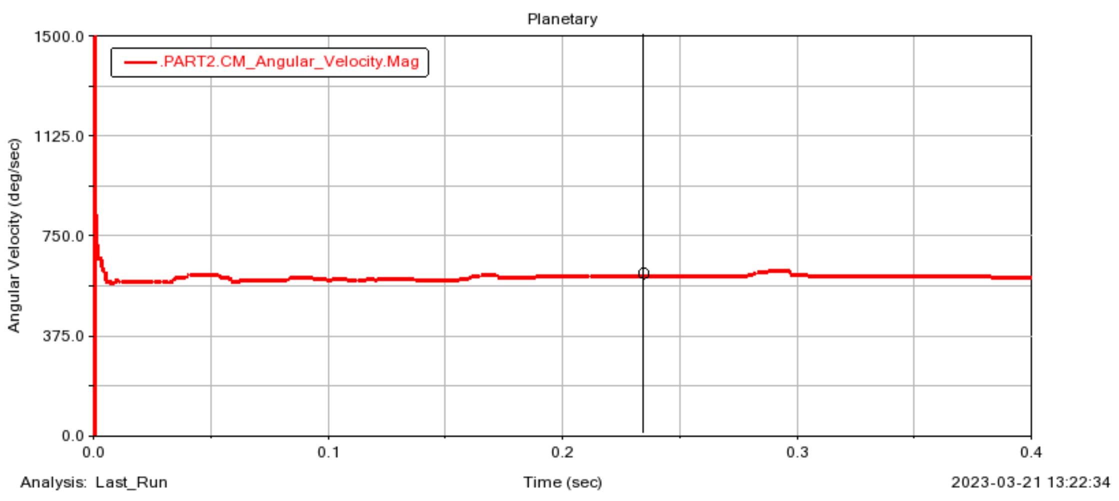


Figure 8: Case 3 Carrier Angular Velocity

The angular velocity of the carrier seems to be more stable than the previous two cases, with much less oscillations. Once again, the plot shows a spike in the beginning of the plot due to the sudden ramp in speed of the Z1 gear. This angular velocity is much less than the previous two cases, with the steady state angular velocity centering around 600 deg/s. This is due to the rotation of both the ring gear and the sun gear. Both the carrier should only be turning if there was an opposite torque applied to the sun gear, creating a differential in speeds between the sun gear and the planet gear. However, in this simulation there was no torque applied to the sun gear, so the velocity of the carrier speed is due to the poor meshing of the gears.

6. Conclusions

This project proved useful in proving the kinematics of the differential planet gear transmission system but came with a lot of complications. The first being proper meshing of the gears. Because the ADAMS Simulation was constantly crashing due to interference in the meshing of the gears, refinement of the gear's teeth width was required to solve the crashing. This in turn created a bad meshing of the gears. The bad meshing, in turn, led to the oscillating plots of the carrier's angular velocity, as well as the carrier moving while no torque was being applied to the sun gear in case 3. The bad meshing of one gearset can compound into the other gears connected to the system, affect all the gears in result. This project taught me the importance of fine-tuning a model without affecting its original purpose. It has also familiarized me with ADAMS simulation software, which was a completely new tool for me, and analyzing the simulation with its design exploration tools. This project also enlightened me to the complexity mechanical systems that seem simple on the outside, but have many components required for its eventual success.

Appendix A: MATLAB Calculations

Term Project Planetary Calculations

Table of Contents

Given Parameters

```
T_sun = 20;      % Sun Gear Teeth
T_Planet = 37;   % Planet Gear Teeth
T_Ring = 94;     % Ring Gear Teeth
T_Z1 = 28;       % Z1 Gear Teeth
T_Z2 = 98;       % Z2 Gear Teeth

Input_Speed = 750*360/60; %deg/s
```

Case 1: Fixed Sun

When the sun gear is fixed, the Z1 gear acts as the input to the system. Z1 Input Gear runs at 750 rpm.

```
S_Z1 = Input_Speed;
S_Z2 = T_Z1/T_Z2 * S_Z1;
S_Ring = S_Z2;
S_Carrier = (T_Ring/T_sun)*S_Ring/(1+T_Ring/T_sun);
display(S_Carrier);
```

```
S_Carrier = 1.0602e+03
```

Case 2: Fixed Ring

When the ring gear is fixed, the sun gear acts as the input to the system. Sun gear runs at 750 rpm.

```
S_Ring = 0;
S_Sun = Input_Speed;
S_Planet = T_sun/T_Planet*S_Sun;
S_Carrier  = S_Planet*T_Planet/T_Ring;
display(S_Carrier);
```

```
S_Carrier = 957.4468
```

Gear Involute Profiles

Sun Gear

```
% INVOLUTE PROFILE
% written by Xi Wu; modified by Andrew Sommer and Nicholas D. Luzuriaga
% DESCRIPTION: Gear parameters are specified, involute profile coordinates
% are sent to a tab delineated text file.

clear all;

% Input parameters for Standard Involute Gear
% diametral pitch Pd = 1/m for English units.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m = 2;                                  % module (mm), or (1/Pd)
z = 20;                                 % number of gear teeth
aDEG = 20;                              % pressure angle on pitch circle (deg)
angleRAD = pi/5;                        % This angle (rad) will determine
                                        % the length of the involute profile

detaA = 0.01;                           % Angular incremental step determines the number
                                        % of points on involute profile.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate parameters from above inputs %%%%%%%%%%%%%%%%%%%%%%
hstar=1;                        % addendum coeff, a constant number for standard gears.
cstar=0.25;                     % clearance coeff, a constant number for standard gears.
a=aDEG*pi/180;                  % pressure angle on pitch circle (rad)

d=m*z;                          % diameter of pitch circle (mm)
da=(z+2*hstar)*m;               % diameter of addendum circle (mm) EXTERNAL GEAR
dd=(z-2*hstar-2*cstar)*m;       % diameter of dedendum circle (mm) EXTERNAL GEAR
db=d*cos(a);                    % diameter of base circle (mm)
s=pi*m/2;                       % tooth thickness on pitch circle (mm)



delta_hstar = 7.55/z;
da_I = (z - 2*hstar + 2*delta_hstar)*m;   % diameter of addendum circle (mm) INTERNAL (
dd_I = (z + 2*hstar + 2*cstar)*m;         % diameter of dedendum circle (mm) INTERNAL (



% Calculate the gear involute profile
alpha=0:detaA:angleRAD;  % pressure angles at different locations on profile (rad)
u=tan(alpha);

x=db*sin(u)/2 - db*u.*cos(u)/2;  % invulte profile equations
y=db*cos(u)/2 + db*u.*sin(u)/2;

% Write coordinates to a text file
gearCO=[x' y' zeros(length(x),1)]; % save coordinates of the points on involute profil
                                   % in matrix format (xi,yi,zi). zi = 0
```

```
save INPUTgsun.txt gearCO -ASCII      % save gearCO as text file

% Calculate half angle of external tooth thickness on base circle (default rot. dir. C
sb_O = cos(a)*(s+m*z*(tan(a)-a));  % external tooth thickness on base circle
AngB_O = (sb_O/db)*180/pi;         % half angle of external tooth thickness on base ci

% Calculate half angle of internal tooth thickness on base circle (default rot. dir. C
sb_I = cos(a)*(s-m*z*(tan(a)-a));  % internal tooth thickness on base circle
AngB_I = (sb_I/db)*180/pi;         % half angle of tooth thickness on base circle (deg

% Print important parameters for CAD software
fprintf('\nEXTERNAL GEAR\n');
```

```
EXTERNAL GEAR
```

```
Rd = dd/2;                    % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd);
```

```
    Rd = 17.500000
```

```
Ra = da/2;                    % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra);
```

```
    Ra = 22.000000
```

```
Rb = db/2;                    % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
    Rb = 18.793852
```

```
Rp = d/2;                     % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
    Rp = 20.000000
```

```

fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_O);
```

```
    Tooth thickness at base circle(Sb): 3.512353
```

```
fprintf('\tRotation of involute from verticle(rot_EXT): %f CCW\n', AngB_O);
```

```
    Rotation of involute from verticle(rot_EXT): 5.353958 CCW
```

```

fprintf('\nINTERNAL GEAR\n');
```

```
INTERNAL GEAR
```

```
Rd_I = dd_I/2;                    % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd_I);
```

```
    Rd = 22.500000
```

```
Ra_I = da_I/2;                    % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra_I);
```

```
    Ra = 18.755000
```

```
Rb = db/2;                      % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
Rb = 18.793852
```

```
Rp = d/2;                       % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
Rp = 20.000000
```

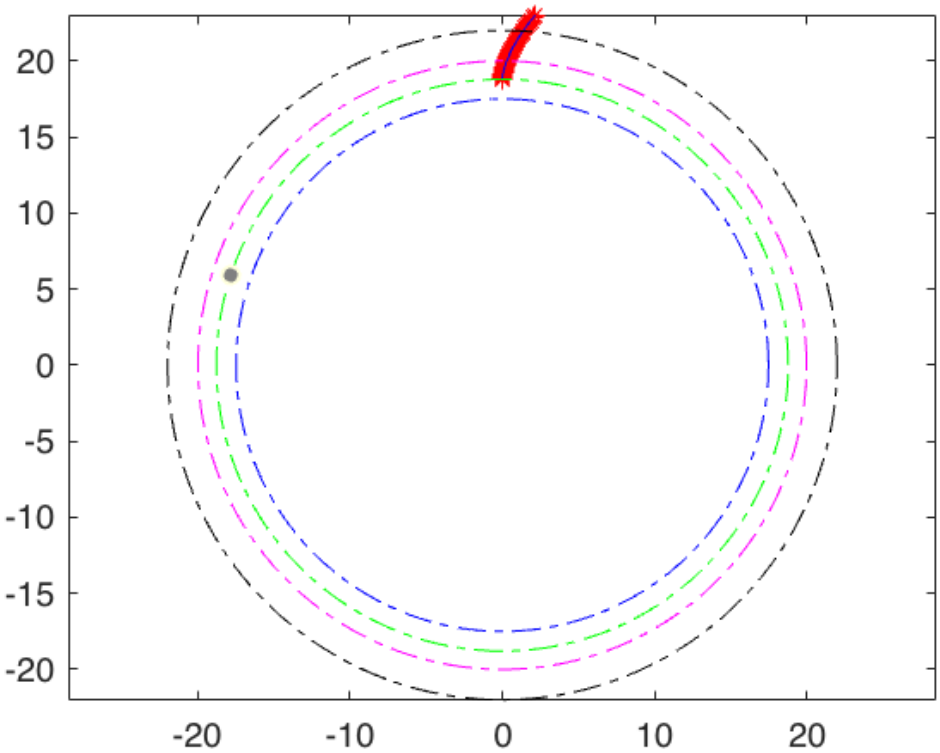
```


fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_I);
```

```
Tooth thickness at base circle(Sb): 2.391910
```

```
fprintf('\tRotation of involute from verticle(rot_INT): %f CW\n\n', AngB_I);
```

```
Rotation of involute from verticle(rot_INT): 3.646042 CW
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% plot involute profile and gear circles
figure(1);
plot(x,y,'r*',x,y,'b-')         % involute profile
hold on

rr = 0:0.001:2*pi;
xxa = (da/2)*cos(rr);           % addendum circle
yya = (da/2)*sin(rr);
plot(xxa,yya,'k-.')

xxp = (d/2)*cos(rr);            % pitch circle
yyp = (d/2)*sin(rr);
plot(xxp,yyp,'m-.')

xxr = (dd/2)*cos(rr);           % dedendum circle
yyr = (dd/2)*sin(rr);
plot(xxr,yyr,'b-.')

xxb = (db/2)*cos(rr);           % base circle
yyb = (db/2)*sin(rr);
plot(xxb,yyb,'g-.')
hold off
axis equal
```



Planet Gear

```
% INVOLUTE PROFILE
% written by Xi Wu; modified by Andrew Sommer and Nicholas D. Luzuriaga
% DESCRIPTION: Gear parameters are specified, involute profile coordinates
% are sent to a tab delineated text file.

clear all;

% Input parameters for Standard Involute Gear
% diametral pitch Pd = 1/m for English units.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m = 2;                                  % module (mm), or (1/Pd)
z = 37;                                 % number of gear teeth
aDEG = 20;                              % pressure angle on pitch circle (deg)
angleRAD = pi/5;                        % This angle (rad) will determine
                                        % the length of the involute profile

detaA = 0.01;                           % Angular incremental step determines the number
                                        % of points on involute profile.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate parameters from above inputs %%%%%%%%%%%%%%%%%%%%%%
hstar=1;                        % addendum coeff, a constant number for standard gears.
cstar=0.25;                     % clearance coeff, a constant number for standard gears.
a=aDEG*pi/180;                  % pressure angle on pitch circle (rad)

d=m*z;                          % diameter of pitch circle (mm)
```

```
da=(z+2*hstar)*m;              % diameter of addendum circle (mm) EXTERNAL GEAR
dd=(z-2*hstar-2*cstar)*m;      % diameter of dedendum circle (mm) EXTERNAL GEAR
db=d*cos(a);                   % diameter of base circle (mm)
s=pi*m/2;                      % tooth thickness on pitch circle (mm)



delta_hstar = 7.55/z;
da_I = (z - 2*hstar + 2*delta_hstar)*m;   % diameter of addendum circle (mm) INTERNAL G
dd_I = (z + 2*hstar + 2*cstar)*m;         % diameter of dedendum circle (mm) INTERNAL G



% Calculate the gear involute profile
alpha=0:detaA:angleRAD;  % pressure angles at different locations on profile (rad)
u=tan(alpha);

x=db*sin(u)/2 - db*u.*cos(u)/2;  % invulte profile equations
y=db*cos(u)/2 + db*u.*sin(u)/2;

% Write coordinates to a text file
gearCO=[x' y' zeros(length(x),1)]; % save coordinates of the points on involute profile
                                   % in matrix format (xi,yi,zi). zi = 0
save INPUTgplanet.txt gearCO -ASCII      % save gearCO as text file

% Calculate half angle of external tooth thickness on base circle (default rot. dir. CC
sb_O = cos(a)*(s+m*z*(tan(a)-a));  % external tooth thickness on base circle
AngB_O = (sb_O/db)*180/pi;         % half angle of external tooth thickness on base ci

% Calculate half angle of internal tooth thickness on base circle (default rot. dir. CW
sb_I = cos(a)*(s-m*z*(tan(a)-a));  % internal tooth thickness on base circle
AngB_I = (sb_I/db)*180/pi;         % half angle of tooth thickness on base circle (deg

% Print important parameters for CAD software
fprintf('\nEXTERNAL GEAR\n');
```

```
EXTERNAL GEAR
```

```
Rd = dd/2;                     % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd);
```

```
    Rd = 34.500000
```

```
Ra = da/2;                     % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra);
```

```
    Ra = 39.000000
```

```
Rb = db/2;                     % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
    Rb = 34.768627
```

```
Rp = d/2;                      % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
	Rp = 37.000000
```

```matlab

fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_O);
```

```
	Tooth thickness at base circle(Sb): 3.988541
```

```matlab
fprintf('\tRotation of involute from verticle(rot_EXT): %f CCW\n', AngB_O);
```

```
	Rotation of involute from verticle(rot_EXT): 3.286391 CCW
```

```matlab

fprintf('\nINTERNAL GEAR\n');
```

```
INTERNAL GEAR
```

```matlab
Rd_I = dd_I/2;                          % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd_I);
```

```
	Rd = 39.500000
```

```matlab
Ra_I = da_I/2;                          % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra_I);
```

```
	Ra = 35.408108
```

```matlab
Rb = db/2;                          % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
	Rb = 34.768627
```

```matlab
Rp = d/2;                           % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
	Rp = 37.000000
```

```matlab

fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_I);
```

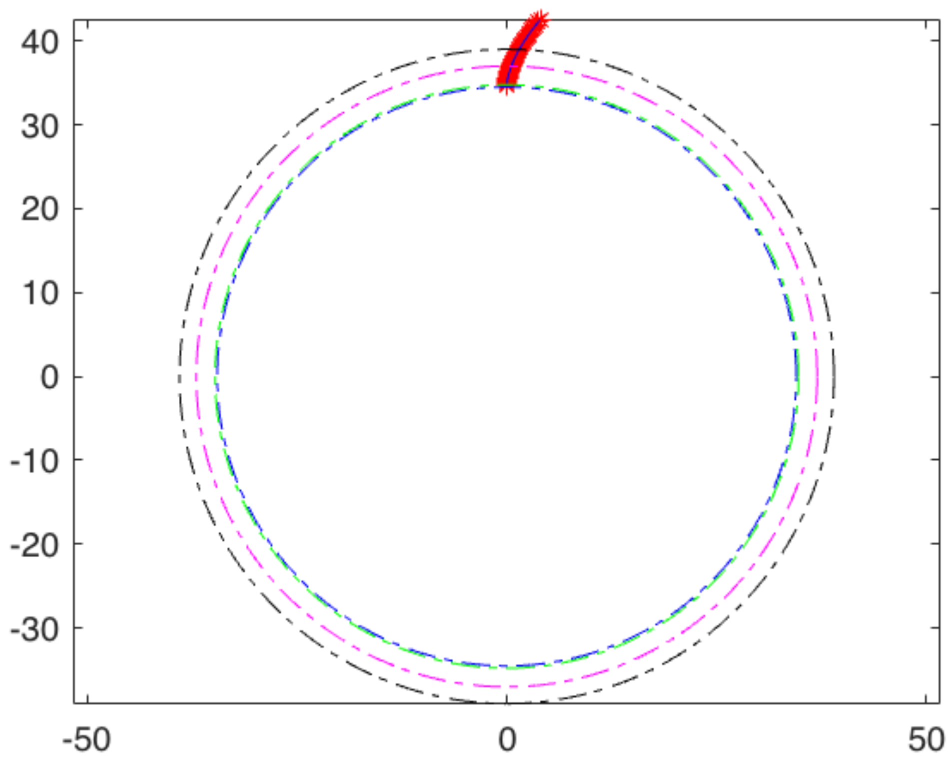
```
	Tooth thickness at base circle(Sb): 1.915722
```

```matlab
fprintf('\tRotation of involute from verticle(rot_INT): %f CW\n\n', AngB_I);
```

```
	Rotation of involute from verticle(rot_INT): 1.578474 CW
```

```matlab

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% plot involute profile and gear circles
figure(2);
plot(x,y,'r*',x,y,'b-')     % involute profile
hold on

rr = 0:0.001:2*pi;
xxa = (da/2)*cos(rr);       % addendum circle
yya = (da/2)*sin(rr);
plot(xxa,yya,'k-.')

xxp = (d/2)*cos(rr);        % pitch circle
```

```
yyp = (d/2)*sin(rr);
plot(xxp,yyp,'m-.')

xxr = (dd/2)*cos(rr);       % dedendum circle
yyr = (dd/2)*sin(rr);
plot(xxr,yyr,'b-.')

xxb = (db/2)*cos(rr);       % base circle
yyb = (db/2)*sin(rr);
plot(xxb,yyb,'g-.')
hold off
axis equal
```



Ring Gear

```
% INVOLUTE PROFILE
% written by Xi Wu; modified by Andrew Sommer and Nicholas D. Luzuriaga
% DESCRIPTION: Gear parameters are specified, involute profile coordinates
% are sent to a tab delineated text file.

clear all;

% Input parameters for Standard Involute Gear
% diametral pitch Pd = 1/m for English units.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m = 2;                                  % module (mm), or (1/Pd)
z = 94;                                 % number of gear teeth
aDEG = 20;                              % pressure angle on pitch circle (deg)
angleRAD = pi/5;                        % This angle (rad) will determine
```

```
                                     % the length of the involute profile

detaA = 0.01;                        % Angular incremental step determines the number
                                     % of points on involute profile.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate parameters from above inputs %%%%%%%%%%%%%%%%%%%%%%
hstar=1;                     % addendum coeff, a constant number for standard gears.
cstar=0.25;                  % clearance coeff, a constant number for standard gears.
a=aDEG*pi/180;               % pressure angle on pitch circle (rad)

d=m*z;                       % diameter of pitch circle (mm)
da=(z+2*hstar)*m;            % diameter of addendum circle (mm) EXTERNAL GEAR
dd=(z-2*hstar-2*cstar)*m;    % diameter of dedendum circle (mm) EXTERNAL GEAR
db=d*cos(a);                 % diameter of base circle (mm)
s=pi*m/2;                    % tooth thickness on pitch circle (mm)



delta_hstar = 7.55/z;
da_I = (z - 2*hstar + 2*delta_hstar)*m;   % diameter of addendum circle (mm) INTERNAL
dd_I = (z + 2*hstar + 2*cstar)*m;         % diameter of dedendum circle (mm) INTERNAL



% Calculate the gear involute profile
alpha=0:detaA:angleRAD;  % pressure angles at different locations on profile (rad)
u=tan(alpha);

x=db*sin(u)/2 - db*u.*cos(u)/2;  % invulte profile equations
y=db*cos(u)/2 + db*u.*sin(u)/2;

% Write coordinates to a text file
gearCO=[x' y' zeros(length(x),1)]; % save coordinates of the points on involute profil
                                   % in matrix format (xi,yi,zi). zi = 0
save INPUTgring.txt gearCO -ASCII      % save gearCO as text file

% Calculate half angle of external tooth thickness on base circle (default rot. dir. C
sb_O = cos(a)*(s+m*z*(tan(a)-a));  % external tooth thickness on base circle
AngB_O = (sb_O/db)*180/pi;         % half angle of external tooth thickness on base ci

% Calculate half angle of internal tooth thickness on base circle (default rot. dir. C
sb_I = cos(a)*(s-m*z*(tan(a)-a));  % internal tooth thickness on base circle
AngB_I = (sb_I/db)*180/pi;         % half angle of tooth thickness on base circle (deg

% Print important parameters for CAD software
fprintf('\nEXTERNAL GEAR\n');
```

```
EXTERNAL GEAR
```

```
Rd = dd/2;                   % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd);
```

```
    Rd = 91.500000
```

```
Ra = da/2;                    % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra);
```

```
Ra = 96.000000
```

```
Rb = db/2;                    % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
Rb = 88.331106
```

```
Rp = d/2;                     % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
Rp = 94.000000
```

```
fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_O);
```

```
Tooth thickness at base circle(Sb): 5.585173
```

```
fprintf('\tRotation of involute from verticle(rot_EXT): %f CCW\n', AngB_O);
```

```
Rotation of involute from verticle(rot_EXT): 1.811405 CCW
```

```
fprintf('\nINTERNAL GEAR\n');
```

```
INTERNAL GEAR
```

```
Rd_I = dd_I/2;                    % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd_I);
```

```
Rd = 96.500000
```

```
Ra_I = da_I/2;                    % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra_I);
```

```
Ra = 92.160638
```

```
Rb = db/2;                    % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
Rb = 88.331106
```

```
Rp = d/2;                     % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
Rp = 94.000000
```

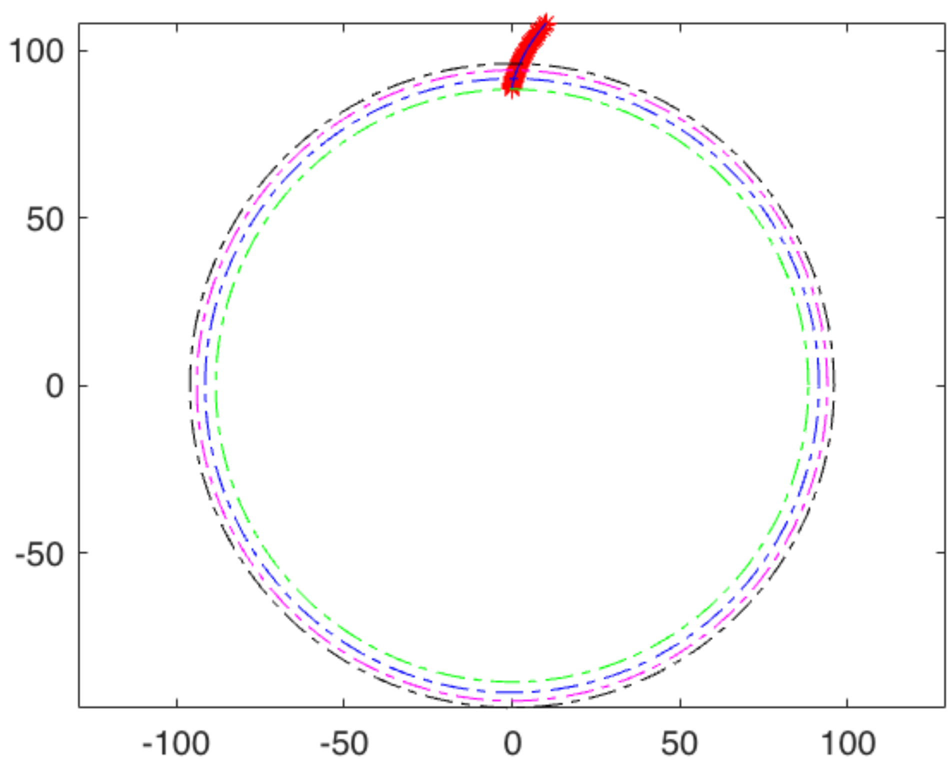
```
fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_I);
```

```
Tooth thickness at base circle(Sb): 0.319090
```

```
fprintf('\tRotation of involute from verticle(rot_INT): %f CW\n\n', AngB_I);
```

```
Rotation of involute from verticle(rot_INT): 0.103489 CW
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% plot involute profile and gear circles
figure(3);
plot(x,y,'r*',x,y,'b-')      % involute profile
hold on

rr = 0:0.001:2*pi;
xxa = (da/2)*cos(rr);        % addendum circle
yya = (da/2)*sin(rr);
plot(xxa,yya,'k-.')

xxp = (d/2)*cos(rr);         % pitch circle
yyp = (d/2)*sin(rr);
plot(xxp,yyp,'m-.')

xxr = (dd/2)*cos(rr);        % dedendum circle
yyr = (dd/2)*sin(rr);
plot(xxr,yyr,'b-.')

xxb = (db/2)*cos(rr);        % base circle
yyb = (db/2)*sin(rr);
plot(xxb,yyb,'g-.')
hold off
axis equal
```



Z1 Gear

```
% INVOLUTE PROFILE
% written by Xi Wu; modified by Andrew Sommer and Nicholas D. Luzuriaga
```

```
% DESCRIPTION: Gear parameters are specified, involute profile coordinates
% are sent to a tab delineated text file.

clear all;

% Input parameters for Standard Involute Gear
% diametral pitch Pd = 1/m for English units.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m = 2;                                   % module (mm), or (1/Pd)
z = 28;                                  % number of gear teeth
aDEG = 20;                               % pressure angle on pitch circle (deg)
angleRAD = pi/5;                         % This angle (rad) will determine
                                         % the length of the involute profile

detaA = 0.01;                            % Angular incremental step determines the number
                                         % of points on involute profile.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate parameters from above inputs %%%%%%%%%%%%%%%%%%%%%%
hstar=1;                         % addendum coeff, a constant number for standard gears.
cstar=0.25;                      % clearance coeff, a constant number for standard gears.
a=aDEG*pi/180;                   % pressure angle on pitch circle (rad)

d=m*z;                           % diameter of pitch circle (mm)
da=(z+2*hstar)*m;                % diameter of addendum circle (mm) EXTERNAL GEAR
dd=(z-2*hstar-2*cstar)*m;        % diameter of dedendum circle (mm) EXTERNAL GEAR
db=d*cos(a);                     % diameter of base circle (mm)
s=pi*m/2;                        % tooth thickness on pitch circle (mm)



delta_hstar = 7.55/z;
da_I = (z - 2*hstar + 2*delta_hstar)*m;   % diameter of addendum circle (mm) INTERNAL G
dd_I = (z + 2*hstar + 2*cstar)*m;         % diameter of dedendum circle (mm) INTERNAL G



% Calculate the gear involute profile
alpha=0:detaA:angleRAD;  % pressure angles at different locations on profile (rad)
u=tan(alpha);

x=db*sin(u)/2 - db*u.*cos(u)/2;  % invulte profile equations
y=db*cos(u)/2 + db*u.*sin(u)/2;

% Write coordinates to a text file
gearCO=[x' y' zeros(length(x),1)]; % save coordinates of the points on involute profile
                                   % in matrix format (xi,yi,zi). zi = 0
save INPUTgz1.txt gearCO -ASCII      % save gearCO as text file

% Calculate half angle of external tooth thickness on base circle (default rot. dir. CC
sb_O = cos(a)*(s+m*z*(tan(a)-a));  % external tooth thickness on base circle
AngB_O = (sb_O/db)*180/pi;         % half angle of external tooth thickness on base ci

% Calculate half angle of internal tooth thickness on base circle (default rot. dir. CW
```

```
sb_I = cos(a)*(s-m*z*(tan(a)-a));  % internal tooth thickness on base circle
AngB_I = (sb_I/db)*180/pi;         % half angle of tooth thickness on base circle (deg

% Print important parameters for CAD software
fprintf('\nEXTERNAL GEAR\n');
```

```
EXTERNAL GEAR
```

```
Rd = dd/2;                     % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd);
```

```
    Rd = 25.500000
```

```
Ra = da/2;                     % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra);
```

```
    Ra = 30.000000
```

```
Rb = db/2;                     % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
    Rb = 26.311393
```

```
Rp = d/2;                      % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
    Rp = 28.000000
```

```

fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_O);
```

```
    Tooth thickness at base circle(Sb): 3.736442
```

```
fprintf('\tRotation of involute from verticle(rot_EXT): %f CCW\n', AngB_O);
```

```
    Rotation of involute from verticle(rot_EXT): 4.068244 CCW
```

```

fprintf('\nINTERNAL GEAR\n');
```

```
INTERNAL GEAR
```

```
Rd_I = dd_I/2;                     % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd_I);
```

```
    Rd = 30.500000
```

```
Ra_I = da_I/2;                     % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra_I);
```

```
    Ra = 26.539286
```

```
Rb = db/2;                     % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
    Rb = 26.311393
```

```
Rp = d/2;                      % Radius of pitch circle (mm)
```

```
fprintf('\tRp = %f\n', Rp);
```

```
Rp = 28.000000
```

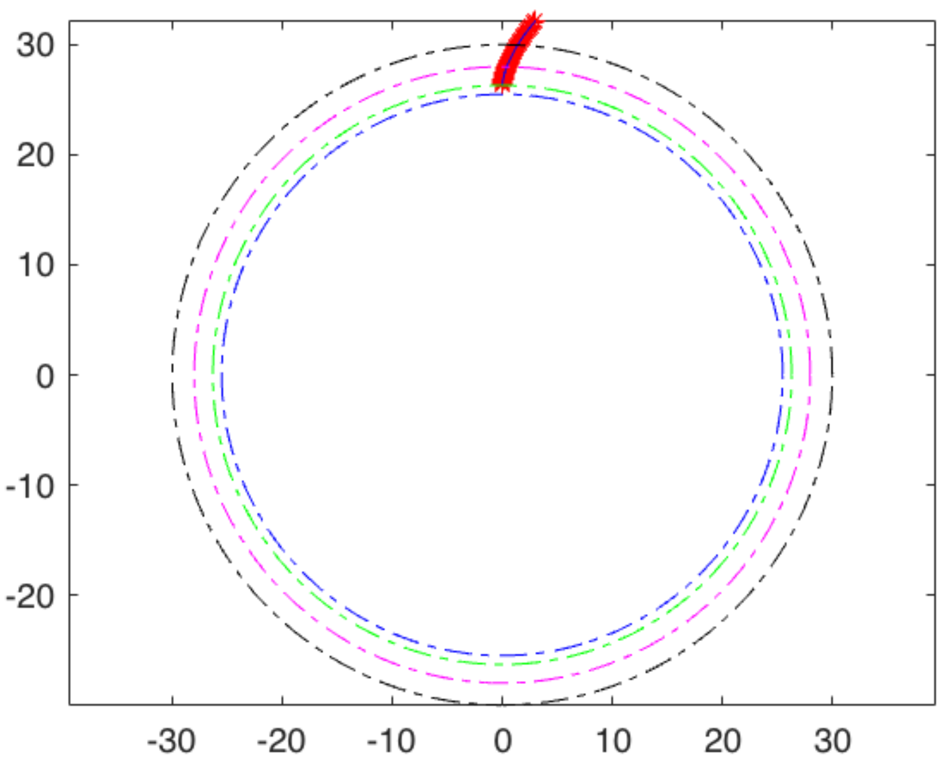
```
fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_I);
```

```
Tooth thickness at base circle(Sb): 2.167821
```

```
fprintf('\tRotation of involute from verticle(rot_INT): %f CW\n\n', AngB_I);
```

```
Rotation of involute from verticle(rot_INT): 2.360327 CW
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% plot involute profile and gear circles
figure(4);
plot(x,y,'r*',x,y,'b-')     % involute profile
hold on

rr = 0:0.001:2*pi;
xxa = (da/2)*cos(rr);       % addendum circle
yya = (da/2)*sin(rr);
plot(xxa,yya,'k-.')

xxp = (d/2)*cos(rr);        % pitch circle
yyp = (d/2)*sin(rr);
plot(xxp,yyp,'m-.')

xxr = (dd/2)*cos(rr);       % dedendum circle
yyr = (dd/2)*sin(rr);
plot(xxr,yyr,'b-.')

xxb = (db/2)*cos(rr);       % base circle
yyb = (db/2)*sin(rr);
plot(xxb,yyb,'g-.')
hold off
axis equal
```



Z2 Gear

```
% INVOLUTE PROFILE
% written by Xi Wu; modified by Andrew Sommer and Nicholas D. Luzuriaga
% DESCRIPTION: Gear parameters are specified, involute profile coordinates
% are sent to a tab delineated text file.

clear all;

% Input parameters for Standard Involute Gear
% diametral pitch Pd = 1/m for English units.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
m = 2;                              % module (mm), or (1/Pd)
z = 98;                             % number of gear teeth
aDEG = 20;                          % pressure angle on pitch circle (deg)
angleRAD = pi/5;                    % This angle (rad) will determine
                                    % the length of the involute profile

detaA = 0.01;                       % Angular incremental step determines the number
                                    % of points on involute profile.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Calculate parameters from above inputs %%%%%%%%%%%%%%%%%%%%%%
hstar=1;                    % addendum coeff, a constant number for standard gears.
cstar=0.25;                 % clearance coeff, a constant number for standard gears.
a=aDEG*pi/180;              % pressure angle on pitch circle (rad)

d=m*z;                      % diameter of pitch circle (mm)
```

```
da=(z+2*hstar)*m;              % diameter of addendum circle (mm) EXTERNAL GEAR
dd=(z-2*hstar-2*cstar)*m;      % diameter of dedendum circle (mm) EXTERNAL GEAR
db=d*cos(a);                   % diameter of base circle (mm)
s=pi*m/2;                      % tooth thickness on pitch circle (mm)



delta_hstar = 7.55/z;
da_I = (z - 2*hstar + 2*delta_hstar)*m;   % diameter of addendum circle (mm) INTERNAL G
dd_I = (z + 2*hstar + 2*cstar)*m;         % diameter of dedendum circle (mm) INTERNAL G



% Calculate the gear involute profile
alpha=0:detaA:angleRAD;  % pressure angles at different locations on profile (rad)
u=tan(alpha);

x=db*sin(u)/2 - db*u.*cos(u)/2;  % invulte profile equations
y=db*cos(u)/2 + db*u.*sin(u)/2;

% Write coordinates to a text file
gearCO=[x' y' zeros(length(x),1)]; % save coordinates of the points on involute profil
                                   % in matrix format (xi,yi,zi). zi = 0
save INPUTgz2.txt gearCO -ASCII      % save gearCO as text file

% Calculate half angle of external tooth thickness on base circle (default rot. dir. C
sb_O = cos(a)*(s+m*z*(tan(a)-a));  % external tooth thickness on base circle
AngB_O = (sb_O/db)*180/pi;         % half angle of external tooth thickness on base ci

% Calculate half angle of internal tooth thickness on base circle (default rot. dir. C
sb_I = cos(a)*(s-m*z*(tan(a)-a));  % internal tooth thickness on base circle
AngB_I = (sb_I/db)*180/pi;         % half angle of tooth thickness on base circle (deg

% Print important parameters for CAD software
fprintf('\nEXTERNAL GEAR\n');
```

```
EXTERNAL GEAR
```

```
Rd = dd/2;                     % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd);
```

```
    Rd = 95.500000
```

```
Ra = da/2;                     % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra);
```

```
    Ra = 100.000000
```

```
Rb = db/2;                     % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
    Rb = 92.089877
```

```
Rp = d/2;                      % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
Rp = 98.000000
```

```
fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_O);
```

```
Tooth thickness at base circle(Sb): 5.697217
```

```
fprintf('\tRotation of involute from verticle(rot_EXT): %f CCW\n', AngB_O);
```

```
Rotation of involute from verticle(rot_EXT): 1.772326 CCW
```

```
fprintf('\nINTERNAL GEAR\n');
```

```
INTERNAL GEAR
```

```
Rd_I = dd_I/2;                    % Radius of dedendum circle (mm)
fprintf('\tRd = %f\n', Rd_I);
```

```
Rd = 100.500000
```

```
Ra_I = da_I/2;                    % Radius of addendum circle (mm)
fprintf('\tRa = %f\n', Ra_I);
```

```
Ra = 96.154082
```

```
Rb = db/2;                    % Radius of base circle (mm)
fprintf('\tRb = %f\n', Rb);
```

```
Rb = 92.089877
```

```
Rp = d/2;                     % Radius of pitch circle (mm)
fprintf('\tRp = %f\n', Rp);
```

```
Rp = 98.000000
```

```
fprintf('\n\tTooth thickness at base circle(Sb): %f\n', sb_I);
```

```
Tooth thickness at base circle(Sb): 0.207046
```

```
fprintf('\tRotation of involute from verticle(rot_INT): %f CW\n\n', AngB_I);
```

```
Rotation of involute from verticle(rot_INT): 0.064409 CW
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% plot involute profile and gear circles
figure(5);
plot(x,y,'r*',x,y,'b-')       % involute profile
hold on

rr = 0:0.001:2*pi;
xxa = (da/2)*cos(rr);         % addendum circle
yya = (da/2)*sin(rr);
plot(xxa,yya,'k-.')

xxp = (d/2)*cos(rr);          % pitch circle
```

```
yyp = (d/2)*sin(rr);
plot(xxp,yyp,'m-.')

xxr = (dd/2)*cos(rr);        % dedendum circle
yyr = (dd/2)*sin(rr);
plot(xxr,yyr,'b-.')

xxb = (db/2)*cos(rr);        % base circle
yyb = (db/2)*sin(rr);
plot(xxb,yyb,'g-.')
hold off
axis equal
```